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Recovery of Compressively Sampled Sparse Signals using Cyclic Matching Pursuit

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***Abstract*—We empirically show how applying a pure greedy algorithm cyclically can recover compressively sampled sparse signals as well as other more computationally complex approaches, such as orthogonal greedy algorithms, iterative thresholding, and ℓ_1-minimization.**

I. Introduction

Under certain conditions, we can recover a vector $\mathbf{x} \in \mathbb{R}^N$ from measurements $\mathbf{u} = \mathbf{\Phi x}$ created by a matrix with unit-norm columns $\mathbf{\Phi} \in \mathbb{R}^{m \times N}$ $(N > m)$. Here we focus on a cyclic application of the pure greedy algorithm matching pursuit (MP) [1]. Given the index set $\Omega_k \subset \Omega = \{1, 2, \ldots, N\}$ (indexing the columns of $\mathbf{\Phi}$), MP augments this set by $\Omega_{k+1} = \Omega_k \cup \{n_k\}$ using

$$n_k = \arg\min_{n \in \Omega} ||\mathbf{r}_k - \langle \mathbf{r}_k, \boldsymbol{\varphi}_n \rangle \boldsymbol{\varphi}_n||_2^2 = \arg\max_{n \in \Omega} |\langle \mathbf{r}_k, \boldsymbol{\varphi}_n \rangle| \quad (1)$$

where $\boldsymbol{\varphi}_n$ is the nth column of $\mathbf{\Phi}$, $\mathbf{r}_k = \mathbf{u} - \mathbf{\Phi x}_k$ is the residual, and the n_k row of $\mathbf{x}_{k+1}$ is defined

$$[\mathbf{x}_{k+1}]_{n_k} = [\mathbf{x}_k]_{n_k} + \langle \mathbf{r}_k, \boldsymbol{\varphi}_{n_k} \rangle. \quad (2)$$

Cyclic MP (CMP) [2], [3] runs as MP at each iteration, but includes a model refinement. Define the ith value of $\Omega_k \subset \Omega = \{1, 2, \ldots, N\}$, $\Omega_k(i)$, $1 \leq i \leq k$. First, CMP finds

$$n_i = \arg\min_{n \in \Omega} ||\mathbf{r}_{k\backslash i} - \langle \mathbf{r}_{k\backslash i}, \boldsymbol{\varphi}_n \rangle \boldsymbol{\varphi}_n||_2^2 = \arg\max_{n \in \Omega} |\langle \mathbf{r}_{k\backslash i}, \boldsymbol{\varphi}_n \rangle| \quad (3)$$

where $\mathbf{r}_{k\backslash i} = \mathbf{u} - \left[\mathbf{\Phi x}_k - \boldsymbol{\varphi}_{\Omega_k(i)}[\mathbf{x}_k]_{\Omega_k(i)}\right]$. Then CMP updates Ω_k such that $\Omega_k(i) = n_i$, and the solution $[\mathbf{x}_k]_{n_i} = \langle \mathbf{r}_{k\backslash i}, \boldsymbol{\varphi}_n \rangle$. Finally, CMP augments Ω_k as in MP, and refines again.

Figure 1 shows the probability of exact recovery ($||\mathbf{x} - \hat{\mathbf{x}}||_2/||\mathbf{x}||_2 < 0.01$) for vectors of varying sparsity k with elements drawn from two distributions, for six undersampling ratios m/N with no noise, using both CMP and Orthogonal MP (OMP). For these experiments, we make $N = 400$, sample $\mathbf{\Phi}$ from the uniform spherical ensemble, and average the results over 100 independent trials for each sparsity and number of measurements. In our implementation, we make CMP run the refinement procedure a max of five times, or until $||\mathbf{r}'_k||_2^2/||\mathbf{r}_k||_2^2 > 0.999$, where $\mathbf{r}'_k$ is the residual after refinement. It is clear that CMP can perform just as well as OMP at this task without matrix inversions. Our final work will include comparisons with other methods, such as iterative thresholding [4], ℓ_1 minimization [5], and two-stage thresholding [6], as well as an analysis of the algorithm.

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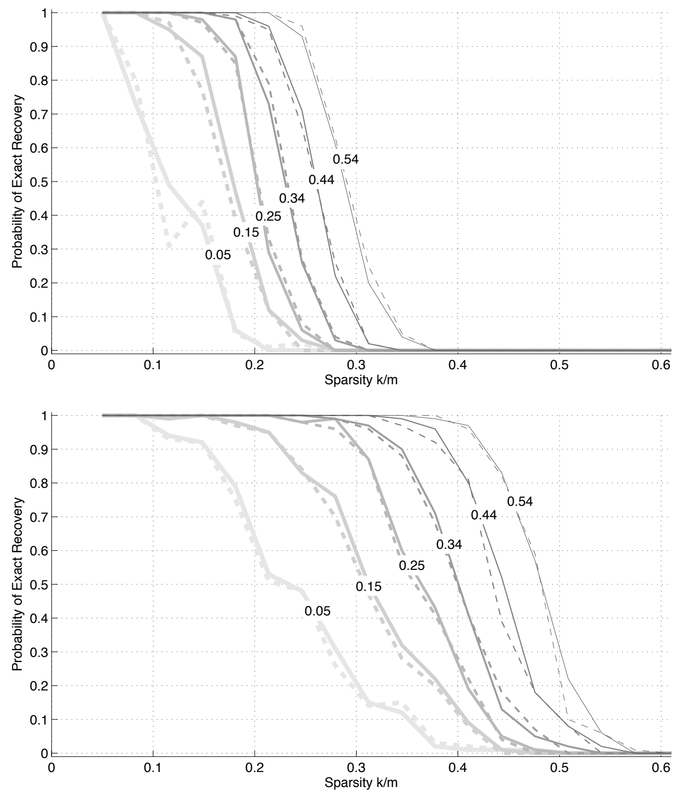

Fig. 1. Probability of exact recovery using CMP (solid) and OMP (dashed) at several undersampling values m/N (labeled). Top: Active elements distributed Constant Amplitude Random Signs [6]. Bottom: Active elements distributed Normal.

References

[1] S. Mallat, *A Wavelet Tour of Signal Processing: The Sparse Way*, 3rd ed. Amsterdam: Academic Press, Elsevier, 2009.

[2] M. G. Christensen and S. H. Jensen, "The cyclic matching pursuit and its application to audio modeling and coding," in *Proc. Asilomar Conf. Signals, Syst., Comput.*, Pacific Grove, CA, Nov. 2007.

[3] B. L. Sturm and M. Christensen, "Cyclic matching pursuit with multiscale time-frequency dictionaries," in *Proc. Asilomar Conf. Signals, Systems, and Computers*, Pacific Grove, CA, Nov. 2010.

[4] T. Blumensath and M. E. Davies, "Normalized iterative hard thresholding: guaranteed stability and performance," *IEEE J. Selected Topics Signal Process.*, vol. 4, no. 2, pp. 298–309, Apr. 2010.

[5] S. S. Chen, D. L. Donoho, and M. A. Saunders, "Atomic decomposition by basis pursuit," *SIAM J. Sci. Comput.*, vol. 20, no. 1, pp. 33–61, Aug. 1998.

[6] A. Maleki and D. L. Donoho, "Optimally tuned iterative reconstruction algorithms for compressed sensing," *IEEE J. Selected Topics in Signal Process.*, vol. 4, no. 2, pp. 330–341, Apr. 2010.